Black holes and gravitational waves in concert — a probe of superstring cosmology

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Abstract

Two strands of observational gravitation, one the search for astrophysical evidence of primordial black holes and the other the search for gravitational waves, may combine to provide strong evidence in favour of cosmological models based on superstring theory, the leading candidate for unifying gravity with the other fundamental forces.

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The quest for a unified theory of the fundamental interactions, including gravity, is the outstanding goal of modern physics. Superstring theory is currently the favoured candidate for such a theory, and as such it should describe the evolution of the very early universe. The primordial spectrum of perturbations generated during that period, provides a means for observationally constraining such a theory, on energy scales that are inaccessible to any form of terrestrial experiment. In this essay, we show that the astrophysical effects of evaporating primordial black holes, together with a stochastic background of primordial gravitational waves, will, if observed, provide strong support for inflationary models within superstring theory [1].

String theory has undergone a revolution in recent years (see [2] for an entertaining review of the most recent developments). It is now widely believed that the five separate perturbative theories are related non–perturbatively by discrete 'duality' symmetries. One such duality is T–duality, relating a theory compactified on a space of large volume with one compactified on a space of small volume. The application of T–duality to

cosmology has recently led to a new inflationary scenario, the so–called *pre–big bang* string cosmology [3].

Inflation is a central paradigm of early universe cosmology [4]. It postulates the existence of a finite, but very rapid, period of accelerated expansion in the universe's distant past. Although inflation was originally developed to explain a number of puzzles of the standard hot big bang model, by far its most important feature is the generation of scalar (density) and tensor (gravitational wave) perturbations from quantum vacuum fluctuations. Small-scale fluctuations generated during inflation are stretched beyond the Hubble radius by the cosmic expansion, where their amplitude remains frozen until they re-enter during the radiation or matter dominated epochs.

A much-advertised prediction of string cosmology is that the spectrum of gravitational waves could be observed by the next generation of gravitational wave detectors, such as the Laser Interferometric Gravitational Wave Observatory (LIGO) currently under construction [5, 6]. In a pre–big bang phase driven by the dilaton field of string theory, the spacetime curvature grows rapidly. As a consequence the spectrum of gravitational wave perturbations grows rapidly towards higher frequencies, scaling as f^3 where f is frequency. The current frequency of these gravitational waves depends on the cosmological model, but reasonable assumptions place the highest frequency f_{s}, corresponding to the horizon scale at the end of the dilaton phase, around the frequencies accessible to LIGO [6]. The potentially high amplitude of these waves is in contrast to conventional models of inflation, where the gravitational wave spectrum must slowly decrease with increasing frequency. In this latter class of models, the microwave background anisotropies detected on large angular scales by the Cosmic Background Explorer (COBE) satellite then leads to an upper limit on the amplitude of the perturbations that is many orders of magnitude below the maximum sensitivity of even advanced versions of the LIGO configuration [7]. The possibility of detecting inflation-generated gravitational waves is therefore a characteristic and distinctive feature of the pre–big bang scenario.

Another striking feature of the pre–big bang scenario is that the gravitational wave amplitude can be related to the probability of black hole formation on a given scale. We define the scalar and tensor perturbations, A_{S}^2 and A_{T}^2, following the conventions of Ref. [8]. The perturbations produced during the dilaton phase of the pre–big bang cosmology are related by the exact equation $A_{\mathrm{T}}^2 = 3A_{\mathrm{S}}^2$ [1]. The energy density of gravitational waves at the present epoch is given in terms of the original amplitude from the expression

$$\Omega_{\mathrm{gw}}(k) = \frac{25}{6}\frac{A_{\mathrm{T}}^2(k)}{z_{\mathrm{eq}}} \tag{1}$$

where $z_{\rm eq} = 24\,000\,\Omega_0 h^2$ is the redshift of matter–radiation equality, Ω_0 and h are the present density and Hubble parameters in the usual units, and k is the wavenumber (interchangeable with frequency f as we set $c = 1$ throughout). This implies that

$$A_{\rm S}^2 = \frac{1}{3} A_{\rm T}^2 = 2 \times 10^{-3} \, \frac{\Omega_{\rm gw}}{10^{-6}} \, \Omega_0 h^2 \tag{2}$$

The advanced LIGO configuration will be sensitive to $\Omega_{\rm gw} \approx 10^{-9}$ over a range of scales around 100 Hz.

Density perturbations on very small scales are constrained, because large inhomogeneities lead to the formation of tiny primordial black holes through immediate gravitational collapse of the perturbations once they enter the horizon. The subsequent Hawking evaporation of these objects results in numerous astrophysical effects. That these effects have yet to be observed places limits on the original number density of black holes that form and, by implication, the amplitude of the original perturbations. Larger black holes will not have evaporated by the present day, but their number density is constrained by their contribution to the overall matter density in the universe.

The criterion for a region to collapse into a black hole during the radiation–dominated epoch is that the density contrast at reentry should exceed some critical value, around $\delta_{\rm c} = 1/3$. The mass of the black hole which forms is comparable to the horizon mass at that time. If a comoving scale f_* reenters the Hubble radius when the temperature is T_*, one can show that

$$\frac{f_*}{f_0} \approx \frac{T_*}{T_{\rm eq}} z_{\rm eq}^{1/2} \tag{3}$$

where $f_0 \approx H_0 \approx 10^{-18}$ Hz is the minimum observable frequency, corresponding to one oscillation in the lifetime of the present Universe. Since $T_{\rm eq} \approx 10^4 T_0 \approx 1{\rm eV}$, it follows that

$$\frac{f_*}{100\,{\rm Hz}} \approx \frac{T_*}{10^9\,{\rm GeV}} \tag{4}$$

The horizon mass at a given temperature is $M_{\rm hor} \approx 10^{32}\,(T/{\rm GeV})^{-2}\,{\rm g}$, yielding a black hole mass for a given mode f_* of

$$M \approx 10^{14} \left(\frac{100\,{\rm Hz}}{f_*} \right)^2 {\rm g} \tag{5}$$

Primordial black holes with initial masses of the order 10^{14}g are at the final stages of their evaporation today. It is intriguing that this mass scale corresponds to frequencies observable by LIGO.

The probability of black hole formation is determined by the dispersion, $\sigma_{\rm hor}$, of the matter distribution smoothed over the length scale $R \approx f^{-1}$ when that scale re-enters the Hubble radius. This dispersion can be obtained directly from the power spectrum $A_{\rm S}^2$ [1], and using Eq. (2) is related to the gravitational wave amplitude by

$$\sigma_{\rm hor}^2 = 3 \times 10^{-3}\, \Omega_0 h^2 \frac{\Omega_{\rm gw}^{\rm s}}{10^{-6}} \tag{6}$$

where $\Omega_{\rm gw}^{\rm s} \equiv \Omega_{\rm gw}(f_{\rm s})$. The fraction of the mass of the universe, β, collapsing into black holes is obtained from the volume of the Universe where the threshold $\delta_{\rm c}$ is exceeded. This is given by the fraction of the gaussian distribution above $\delta_{\rm c}$, which corresponds to $\beta = {\rm erfc}(\delta_{\rm c}/\sqrt{2}\,\sigma_{\rm hor})$, where 'erfc' is the complementary error function.

Black holes with mass greater than 10^9g evaporate after one second and the observational constraints are well known [9]. Only a tiny fraction of the mass of the universe may form primordial black holes; a robust upper limit on the allowed initial mass fraction is $\beta(M) < 10^{-20}$ and this implies that $\sigma_{\rm hor} < 0.04$. Saturating the observational bound on $\sigma_{\rm hor}$ gives, from Eq. (6), the amplitude of gravitational waves at frequency $f_{\rm s}$ that would lead to astrophysical effects from primordial black holes:

$$\Omega_{\rm gw}^{\rm s} = \frac{5 \times 10^{-6}}{\Omega_0 h^2} \tag{7}$$

This is below the present tightest bound on the gravitational wave background, which comes from its effect on nucleosynthesis and requires $\Omega_{\rm gw} < 5 \times 10^{-5}$. But it is well within the sensitivity of the advanced LIGO detectors [6].

We stress that a number of assumptions go into this result, discussed in detail in Ref. [1]. The peak of the perturbation spectra arising from the pre–big bang scenario is assumed to lie in the frequency range accessible to LIGO. In general this need not be so, and could lead to a much reduced level of both gravitational waves and density perturbations on these scales. The crucial point though is that the amplitudes are so closely linked, and if one of the two spectra is observable there is reasonable hope that the other will be too. For example, if gravitational waves are detected at around the level of Eq. (7), then string cosmology predicts that black holes be observable. Detection of a gravitational wave background above the level indicated in Eq. (7), without black hole detection, would suggest that these gravitational waves could not have been generated by the dilaton phase of string cosmology. Detection of the two in concert, with the correct relation between their amplitudes, would provide possibly the first observational evidence for string theory.

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